Course Hand Out
Subject Name: DISCRETE MATHEMATICS
Prepared by: Mr. K. VAMSHEE KRISHNA, Assistant Professor, CSE
Year, Semester, Regulation: II Year- II Sem (R18)

UNIT – I (SET , RELATION, FUNCTION)

➤ **KEY POINTS**

Basic mathematics on the real numbers Why: You need to be able to understand, write, and prove equations and inequalities involving real numbers.
• Standard functions and their properties: addition, multiplication, exponentiation, logarithms.
• More specialized functions that come up in algorithm analysis: floor, ceiling, max, min.
• Techniques for proving inequalities, including: – General inequality axioms (transitivity, anti-symmetry, etc.) – Inequality axioms for R (i.e., how < interacts with addition, multiplication, etc.) – Techniques involving derivatives (assumes calculus) (*):
∗ Finding local extrema of f by solving for f 0 (x) = 0. (*)
 ∗ Using f 00 to distinguish local minima from local maxima. (*)
 ∗ Using f 0 (x) ≤ g 0 (x) in [a, b] and f(a) ≤ g(a) or f(b) ≤ g(b) to show f(x) ≤ g(x) in [a, b]. (*)
 • Special subsets of the real number: rationals, integers, natural numbers.
1.4.3 Fundamental mathematical objects
Why: These are the mathematical equivalent of data structures, the way
that more complex objects are represented.
• Set theory.
– Naive set theory.
– Predicates vs sets.
– Set operations.
– Set comprehension.
– Russell's paradox and axiomatic set theory.
• Functions.
– Functions as sets.
– Injections, surjections, and bijections.
– Cardinality.
– Finite vs infinite sets.
– Sequences.
• Relations.
– Equivalence relations. Equivalence classes and quotients.
– Orders: total orders, partial orders, lattics, and well orders. Order
types and ordinals.
• The basic number tower.
– Countable universes: N, Z, Q. (Can be represented in a computer.)
– Uncountable universes: R, C. (Can only be approximated in a
computer.)
• Other algebras.
– The string monoid. (*)
– Zm and Zp.
– Polynomials over various rings and fields.
2MARKS QUESTIONS

1. Draw venn diagram showing: (i) AÈB Ì AÈC but BËC, (ii) AÇB Ì AÇC but BËC.

2. Let A = {1, 2, 3, 4}, and R is a relation defined by "a divides b". Write R as a set of ordered pair, draw directed graph. Also find R^{-1}

3. Let R be a binary relation defined as R = {(a, b) Î R^2: a-b < 3}, determine whether R is reflexive, symmetric and transitive.

4. Let A = {1, 2, 3, 4, 5, 6}, construct pictorial description of the relation R on A defined as R = {(a, b): $(a-b)^2$ Î A}.

5. Let A = {1, 2, 3, 4}, give an example of a mapping which is (i) neither symmetric nor anti-symmetric, (ii) anti-symmetric and reflexive but not transitive, (iii) transitive and reflexive but nit anti-symmetric.

7. Let R be a relation on the set A = {a, b, c} defined by R = {(a, b), (b, c), (d, c), (d, a), (a, d), (d, d)}. Write the relation matrix of R and find (i) reflexive closure of R,
(ii) symmetric closure of R and (iii) transitive closure of R.

8. Show that a relation R defined on the set of real numbers as (a, b) R (c, d) iff $a^2 + b^2 = c^2 + d^2$. Show that R is an equivalence relation.

9. An inventory consists of a list of 115 items, each marked "available" of "unavailable". There are 60 available items. Show that there are at least two available items in the list exactly four items apart.

10. List all possible functions from A to B, A = {a, b, c}, B = {0, 1}. Also indicate in each case whether the function is one-to-one, is onto and one-to-one-onto.

10 MARKS QUESTIONS

11. Let A = {1, 2, 3}, B = {p, q} and C = {a, b}. Let f: A → B is f = {(1, p), (2, p), (3, a)} and g: B → C is given by {(p, b), (q, b)}. Find gof and show it pictorially.

12. If f is function from A to B and g is function B to C and both f and g are onto. Show that gof is also onto. Is gof one-to-one if both f and g are one-to-one.

13. Let f, g and h: R → R be defined by (R is the set of real numbers)

$$f(x) = x + 2, \quad g(x) = (1 + x^2)^{-1}, \quad h(x) = 3.$$

Compute $f^{-1}g(x)$ and $hf (g f^{-1}) (hf(x))$.

14. Show that the function f and g both of which are from N ´ N to N given by $f(x, y) = x + y$ and $g(x, y) = xy$ are onto but not one-one.

15. Show that the function $f(x) = k$, where k is a constant, is primitive recursive.

16. State and prove pigeonhole principle.

2. UNIT – II (PROPOSITIONAL LOGIC)

➢ KEY POINTS

Propositional logic Propositional logic is the simplest form of logic. Here the only statements that are considered are propositions, which contain no variables. Because propositions contain no variables, they are either always true or always false. Examples of propositions:
• 2 + 2 = 4. (Always true).

• 2 + 2 = 5. (Always false).

Examples of non-propositions:

• x + 2 = 4. (May be true, may not be true; it depends on the value of x.)

• x · 0 = 0. (Always true, but it's still not a proposition because of the variable.)

• x · 0 = 1. (Always false, but not a proposition because of the variable.)

2MARKS QUESTIONS

1. Make a truth table for the following:
   (i) (pÚq) Ù r   (ii) (pÚ~q) Þ r   (iii) (p ¯ q) Ù (p ¯ r)

2. State and prove De Morgan's law for logic.

3. Is ((pÚ~q) Ù(~pÚ~q)) Úq a tautology?

4. The pierce arrow ¯ (NOR) is a logical operation defined as p ¯ q ° ~ (pÚq), Prove that
   (i) ~ p ° p ¯ q and (ii) (pÙq) ° (p ¯ p) ¯ (q ¯ q).

5. Prove the following: (i) p Ú (~p Ù q) ° (pÚq), (ii) p Ù (~pÚq) ° (p Ù q)

6. Consider the following conditional statement:

   If the flood destroy my house or the fires destroy my house, then my insurance company will pay me.

   Write the converse, inverse and contrapositive of the statement.

7. Given the following statements as premises, all referring to an arbitrary meal:
   If he takes coffee, he does not drink milk.
   He eats crackers only if he drinks milk.
   He does not take soup unless he eats crackers.
   At noon today, he had coffee.

   Whether he took soup at noon today? If so what is the correct conclusion.

8. There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying the same thing?

9. Is the following argument valid?
   If taxes are lowered, then income rise
   Income rise
   \Taxes are lowered

10 MARKS QUESTIONS

10. Write the following statement in symbolic form using quantifiers:
    (i) All students have taken a course in mathematics.
    (ii) Some students are intelligent, but not hardworking.

    11. Let p(x): x is mammal and q (x): x is animal. Translate the following in
    English: ("x)(q(x) Ù(~p(x)))

12. Let A = {1, 2, 3, 4, 5}, determine the truth value of the following:
    (i) ("xÎA)(x + 3 = 10),   (ii) ($xÎA)(x + 3 <5).

13. Write the negation of the following statement: $
    x Î R x > 3 Þ x^2 > 9

14. Prove the following or provide a counter example:

$$A \cap B \cap \overline{A} \cap \overline{B} \cap \overline{A} = B$$

15. Prove or disprove the statement that if x and y are real numbers: $(x^2 = y^2) \cup (x = y)$.

16. Let n be an integer, prove that n^2 is an odd then n is odd.

17. Use the method of contradiction to prove that $\ddot{O}5$ is not a rational number.

3. UNIT – III (COMBINOTRICS)

2MARKS QUESTIONS

1. Find the reccurence relation with initial condition for the following:
   (i) 2, 10, 50, 250, ……. (ii) 1, 1, 3, 5, 8. 13. 21, …..

2. Solve $a_n - 3 a_{n-1} = 2$, $n^3 2$, with $a_0 = 1$.

3. Solve $a_n - 2 a_{n-1} - 3 a_{n-2} = 0$, $n^3 2$, with $a_0 = 3$, $a_1 = 1$.

4. Solve $a_{n+2} - 2 a_{n+1} + a_n = 2^n$; $a_0 = 2$, $a_1 = 1$.

5. Solve the following using the initial condition as $s(0) = s(1) = 1$
   $s(k) - 9 s(k-1) + 8 s(k-2) = 9k + 1$.

6. Determine the generating function of the following numeric function
   $a_n = 2^n$, if n is even
   $= -2^n$, if n is odd

7. Find the closed form for the generating function for the following
   (i) 1, 0, -1, 0, 1, 0, -1, 0, 1, …..
   (ii) 0,3,3,3,3,…….

8. Use induction to that
   (i) $2 + 4 + 6 + …. + 2n = n^2 + n$, for $n^3 1$
   (ii) $11^n - 4^n$ is divisible by 7, for $n^3 1$
   (iii) $2n > n^2$, for $n^3 5$

9. What is the number of solutions of the equation $x + y + z + w = 20$, if x, y, z and w are non-negative integers.

10 MARKS QUESTIONS

10. How many solutions are there to the equation $a = b + c + d + e + f = 21$, where each variable is non-negative integer such that (i) $1 £ x$, (ii) all variables are $^3 2$.

11. Prove using counting argument $C (n, r) = C (n-1, r) + C (n-1, r-1)$.

12. Find the generating function to select 10 candy bars from large supplies of six different kind.

13. Find the generating function for the number of ways to select (with repetition allowed) r objects from a collection of n distinct objects.

14. In how many different ways can eight identical balls be distributed among three children if each receives at least two balls and no more than four balls?

15. Find the number of ways that 9 students can be seated in the room so that there is at least one student in each of the five rows.

UNIT – IV (ALGEBRAIC STRUCURE

2MARKS QUESTIONS

1. Let A = {a, b}. Which of the following tables define a semi- group? Which define monoid on A

```
* | a  b          * | a   b          * | a  b
-----            -----            -----
a | b  a          a | b   b          a | a  b

b | a  b          b | a   a          b | b  a
```

2. Let (A, *) be a semi group, further more for every a and b in A, if a ≠ b, then

(i) Show that for every a in A $a * a = a$

(ii) Show that for every a, b in A $a * b * a = a$ 　　　　　　　　　　　　　$a * b \, ^1 \, b * a$

(iii) Show that for every a, b, c in A $a * b * c = a * c$

3. Differentiate between semigroup and subgroup with example.

4. Define a group. Let S = {0, 1, 2, 3, 4, 5, 6, 7} & * denote "multiplication modulo 8 i.e. $x * y = (xy)\mathrm{mod}8$.

5. Let G be the set of all non zero real no's & let $a * b = \dfrac{ab}{2}$ Show that (G, *) is an abelian group.

6. Let G be a group & a & b be the elements of G. Then prove that

(i) $\left(a^{-}_{1}\right)^{-1} = a$ 　　　　　　　　　　(ii) $(ab)^{-1} = b^{-1}a^{-1}$

10 MARKS QUESTIONS

7. Show that G = $\{1,-1,i,-i\}$ where $i = \sqrt{-1}$ is an abelian group with respect to multiplication as a binary operation.

8. Let G_1 and G_2 be subgroup of a group G

(i) Show that $G_1 \, Ç \, G_2$ is also a subgroup of G

(ii) Is $G_1 \, È \, G_2$ is always a subgroup of G.

9. If a cyclic group G is generated by an element a of order n, then a^n is a genertor of G, iff the greatest divisor of m & n is 1, i.e. iff m & n are relatively prime.

10. Show that the group {1,2,3,4,5,6 X_7) is a cyclic group. How many generators are there.

11. Define permutation group. Let A = $\{1,2,3,4,5\}$. Find $(13)o\,(245)o\,(23)$

$$\begin{pmatrix} 2 & 3 & 4ü \\ & & ý \end{pmatrix}$$ as a product of transposition and hence

$$\begin{pmatrix} 2 & 4 & 3þ \end{pmatrix}$$

12. Express the permutation A= í
$$\hat{1}^{1}$$
determine whether it is even or odd.

13. What do you mean by group isomorphism. Give example with example.

14. Let g be a group & let $g \in G$. Define a function $g\hat{\ } : G \to G$ by $g\hat{\ }(x) = gxg^{-1}$. Show that

(i) $g\hat{\ }$ is a homomorphism

(ii) $g\hat{\ }$ is a One to One (iii)

$g\hat{\ }$ is Onto.

15. Let G be the group of real no.'s under addition 7 let $G^{/}$ be the group of positive real no's under multiplication. Prove that mapping $f : G \circledR G^{/}$ defined by $f(a) = 2^a$ is homomorphism.

16. Show that the additive group Z_4 is isomorphic to the multiplicative group of non zero element of Z_5.

17. If (R, +) is a ring with unity, then show that, for all a $\in R$

(i) (-1).a = -a
(ii) (-1).(-1) = 1

18. If R is a ring such that a^2 = a $\forall$ a $\in$R Prove that

(i) a + a = 0 $\forall$ a $\in$R i.e. each element of R is its own additive inverse
(ii) a + b =0 Þ a = b
(iii) R is a commutative ring.

19. Show that the set R of real no. with composition o & * by $a \circ b = a + b + 1$ & $a * b = ab + a + b$ is a ring. Determine 0 element 7 1- element of the ring.

20. Show that in a field F

(i) $(-a)b = -(ab)$, a,b Î F (ii) $(-a)(-b)^{-1} = ab^{-1}$ (iii) $(-a^{-1})^{-1} = a$

21. Let S be a set of all 2 X2 matrices of the form $\begin{matrix} é0 & 0ù \\ ê & ú \end{matrix}$ where a, b are integers. Show that S

$$ë a \quad b û$$
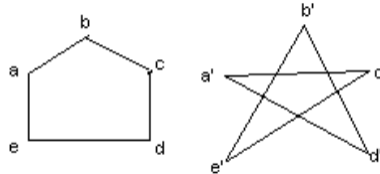
is a ring 7 not a field.

5. UNIT – V (GRAPH THEORY & AUTOMATA)

2 MARKS QUESTIONS

1. What do you mean by graph isomorphism, show it by example?
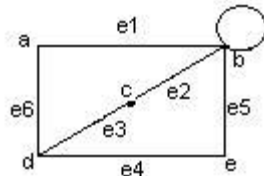
2. Show that the given graph are isomorphic.

3. Define walk, path & trial and also from the given graph find which of the following sequences paths, simple paths, are cycle and simple cycle.

(a) be_7b

(b) $de_3ce_2be_5ee_4d$

(c) $ae_6de_3ce_2be_5e$

4. Prove that if the graph has n vertices and vertex 'u' is connected to vertex 'w' then there exist a path from u to w of length no more than n.

5. Prove that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.

6. Define Eulerian Graph and prove that a non empty connected graph G is Eulerian iff its vertices are all of even degree.

10 MARKS QUESTIONS

7. Differentiate between Eulerian graph & Hamiltonian graph with example.

8. (i) Determine whether the following graph contain Eulerian circuit. If it does, then find an Eulerian circuit.

9. Determine whether the given graph has Hamiltonian circuit. If it does, find such a circuit.
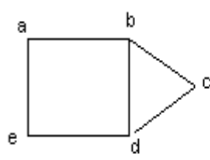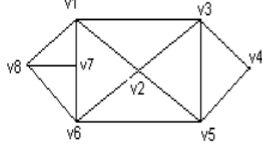
10. Prove that if G is connected graph with n vertices & (n-1) edges then G is a tree.
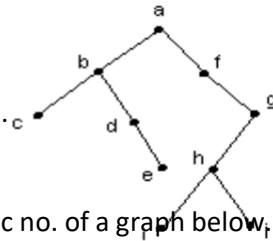
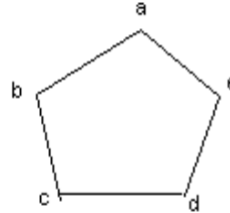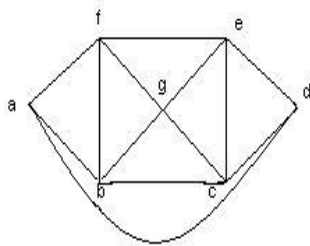11. A simple graph G has a spanning tree iff G is connected.

12. Find the In Order, Pre Order, & Post Order of Binary tree.

13. Construct a binary tree whose in order 7 pre order traversal is given.
 In Order   d g b e i h j a c f   Pre Order a b d g e h I j c f

14. Write a short notes on chromatic number & hence find the chromatic no. of a graph below.

15. Prove that the chromatic number of a tree is always 2 & chromatic polynomial is $\lambda(\lambda-1)^{n-1}$

16. Prove these are equivalent

(i) A graph G is 2- colourable.

(ii) G is bipartite

(iii) Every cycle of G is of even length.

17. Draw the transition diagram of a finite state automation that accept the set of strings over {0, 1} that contain an even no. of 0's & an odd no. of 1's.

18. Write regular expressions for the following languages

(i) The set of strings over an alphabet {a,b,c} containing atleast one a & atleast one b.

(ii) The set of strings of 0's & 1's with atmost one pair of consecutive 1's.

(iii) The set of all strings of 0's & 1's s.t every pair of adjacent 0's appear before any pair of adjacent 1's.

(iv) The set of strings of 0's & 1's whose no. of 0's is divisible by 5.

19. Write regular expression for the following languages over {0,1}

(i) All strings ending in 01

(ii) All strings containing an even no's of 0's

(iii) All string having atleast two occurrence of the substring 00.

7